PROPAGATION OF A TURBULENT FAN JET ALONG PLANE AND CONCAVE SURFACES

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 5, pp. 909–914, 1968

UDC 532.517.4

The flows associated with the propagation of a fan-shaped turbulent jet along plane and concave surfaces are theoretically and experimentally investigated.

The special apparatus used to investigate the flow of a semibounded fan jet along plane and concave surfaces is shown schematically in Fig. 1.

The air forming the jet flowed from the variable annular nozzle created between a metal plate and a cap with r_0 = 25 mm.

To investigate the flow over a concave surface we used three hemispheres with radius of curvature R = = 60, 80, and 120 mm, respectively. The inner face of the cap was adjusted to keep the sides of the nozzle parallel in each case, the gap b_0 being maintained constant and equal to 1.78 mm.

The nozzle exit velocity was also constant at 47.5 m/sec.

The velocity profile was recorded with a special three-tube array of hypodermic needles with a 0.2 mm intake opening and a lateral tube cutoff angle of 75°. This array was secured to a traverse system with three degrees of freedom rigidly attached to the metal plate.

In investigating the flow over a concave surface, it was possible to displace the tube array radially correct to 0.5 mm. The transverse coordinate could be read with an accuracy of 0.05 mm.

Before each experiment, the metal plate was strictly leveled.

The velocity in the jet was determined with an MMN inclined micromanometer graduated in 0.2 mm H_2O.

An analysis of the experimental data showed that in the outer region of a semibounded fan jet propagating

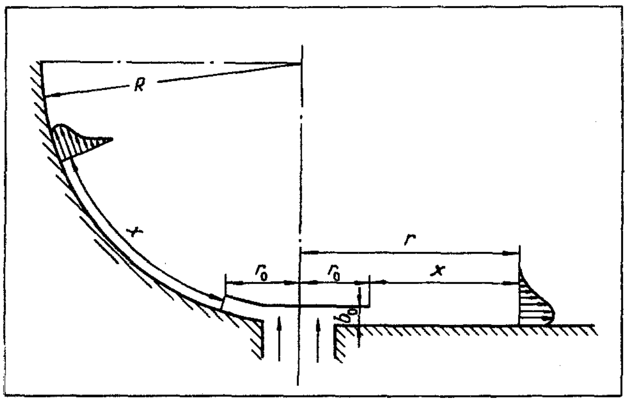

Fig. 1. Diagram of the experimental apparatus.

over a plane or a concave surface the profile of the longitudinal velocity component can be approximated by the known universal profile of free turbulent jet flows

$$\frac{u}{u_m} = \mathrm{sch}^2\, 0.88\, \frac{y-\delta_m}{\delta}. \qquad (1)$$

The velocity profile in the wall region is well described by a logarithmic relation of the type

$$\frac{u}{u_m} = 1 - 0.18 \log \frac{y}{\delta_m}. \qquad (2)$$

The experimental data on δ and δ_m as functions of the distance x are plotted in Fig. 2. The functional relation between these quantities and the distance x can be described by linear relations of the type

$$\frac{\delta}{b_0} = \frac{0.0545\,R}{R+23.4}\,\frac{x}{b_0} + \frac{0.616\,R}{R-5},$$
$$\frac{\delta_m}{b_0} = \frac{0.0128\,R}{R+23.4}\,\frac{x}{b_0} + \frac{0.149\,R}{R-5}. \qquad (3)$$

The good agreement between the experimental data and Eqs. (3) indicates the reliability of this method of determining the geometric characteristics of the jet.

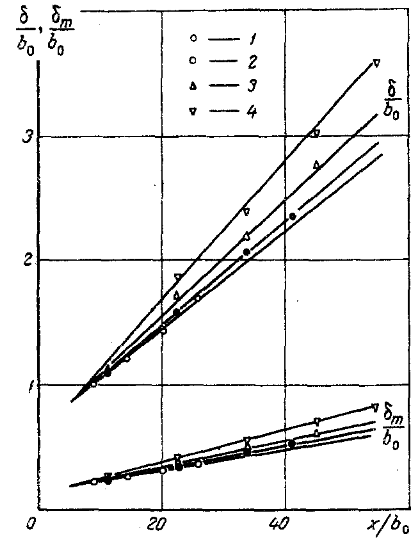

Fig. 2. Thicknesses δ and δ_m as functions of x for a concave surface (1–R = 60; 2–80; 3–120) and a plane surface (4). The straight lines have been plotted on the basis of Eqs. (3).

The width of the jet is determined from Eq. (1) on the assumption that, at the jet boundary, the longitudinal velocity component is 1% of its maximum value. Using (3), we obtain for the width of a fan jet flowing over a concave surface of any radius of curvature,

$$\frac{b}{b_0} = \frac{0.198\,R}{R+23.4}\,\frac{x}{b_0} + \frac{2.244\,R}{R-5}. \tag{4}$$

The concavity of the surface has a marked influence on the geometric characteristics of the jet. Not only does the line of maximum velocities approach the surface as its radius of curvature decreases, but at the same time a general contraction of the jet is distinctly observed. This possibility was pointed out in connection with a plane semibounded jet in [1], where it was concluded that the thickness of the jet would be substantially reduced on a concave as compared with a plane surface.

Using the experimental results, we can calculate the velocity, temperature, and flow rate characteristics of this type of flow.

At constant pressure in the jet and a uniform velocity and temperature profile at the outlet of the annular nozzle (Fig. 1), assuming that the forces of friction against the wall are insignificant [2], we write the law of conservation of momentum in the form

$$2\pi\rho_0 r_0 b_0 u_0^2 = 2\pi r\int_0^\infty \rho u^2\,dy\,f(R,\ r), \tag{5}$$

while the constancy of the jet enthalpy, calculated from the excess temperatures, is expressed by the relation

$$2\pi\rho_0 r_0 b_0 u_0\,\Delta T_0 = 2\pi r\int_0^\infty \rho u\,\Delta T\,dy\,f(R,\ r), \tag{6}$$

where $f(R, r) = R\sin 57.3\,(r/R)/r$ is a function taking into account the decrease in the length of the circumference at an equal distance r from the geometric axis of the jet for flow over a concave surface as compared with flow over a plane surface.

Taking $\rho_0 = \rho$ and carrying out transformations, we obtain

$$\frac{u_m^2}{u_0^2} = b_0\frac{r_0}{r}\left(\int_0^\infty \frac{u^2}{u_m^2}\,dy\right)^{-1}\frac{1}{f(R,\ r)}, \tag{7}$$

$$\frac{u_m}{u_0}\,\frac{\Delta T_m}{\Delta T_0} = b_0\frac{r_0}{r}\left(\int_0^\infty \frac{u}{u_m}\,\frac{\Delta T}{\Delta T_m}\,dy\right)^{-1}\frac{1}{f(R,\ r)}. \tag{8}$$

Assuming [2] that $\Delta T/\Delta T_m = (u/u_m)^{1/2}$ and using (1), (2), we evaluate the integral in (7) and (8):

$$\int_0^\infty \frac{u^2}{u_m^2}\,dy = \int_0^{\delta_m}\left(1-0.18\log\frac{y}{\delta_m}\right)^2 dy +$$

$$+\int_{\delta_m}^\infty \mathrm{sch}^4\,0.88\,\frac{y-\delta_m}{\delta}\,dy = 0.855\,\delta_m + 0.758\,\delta, \tag{9}$$

$$\int_0^\infty \frac{u}{u_m}\,\frac{\Delta T}{\Delta T_m}\,dy = \int_0^{\delta_m}\left(1-0.18\log\frac{y}{\delta_m}\right)^{3/2} dy +$$

$$+\int_{\delta_m}^\infty \mathrm{sch}^3\,0.88\,\frac{y-\delta_m}{\delta}\,dy = 0.888\,\delta_m + 0.893\,\delta. \tag{10}$$

Substituting solutions (9) and (10) into (7) and (8), respectively, and using (3), we obtain equations for the change of maximum velocity and maximum excess temperature for flow over a plane surface and a concave surface of any radius of curvature:

$$\frac{u_m}{u_0} = (b_0 r_0)^{1/2}\times$$

$$\times\left[\left(\frac{0.0522\,R}{R+23.4}\,x + \frac{0.595\,R}{R-5}\,b_0\right)R\sin 57.3\,\frac{r}{R}\right]^{-1/2} \tag{11}$$

$$\frac{\Delta T_m}{\Delta T_0} = (b_0 r_0)\times$$

$$\times\left[\frac{u_m}{u_0}\left(\frac{0.06\,R}{R+23.4}\,x + \frac{0.683\,R}{R-5}\,b_0\right)R\sin 57.3\,\frac{r}{R}\right]^{-1} \tag{12}$$

Replacing the ratio u_m/u_0 in (12) by its value from (11), we finally obtain

$$\frac{\Delta T_m}{\Delta T_0} = \left(0.93\sqrt{b_0 r_0}\right)\times$$

$$\times\left[\left(\frac{0.06\,R}{R+23.4}\,x + \frac{0.683\,R}{R-5}\,b_0\right)R\sin 57.3\,\frac{r}{R}\right]^{-1/2} \tag{13}$$

From a comparison of (13) and (11), it follows that there is a proportional relationship between the dimensionless velocity and the temperature in each cross section of the jet:

$$\frac{\Delta T_m}{\Delta T_0} = 0.868\,\frac{u_m}{u_0}. \tag{14}$$

In Fig. 3 the experimental and calculated data for the fall of maximum velocity along the surface are in satisfactory agreement.

The volume flow rate in a cross section of the semibounded fan jet is given by

$$Q = 2\pi r\int_0^\infty u\,dy\,f(R,\ r). \tag{15}$$

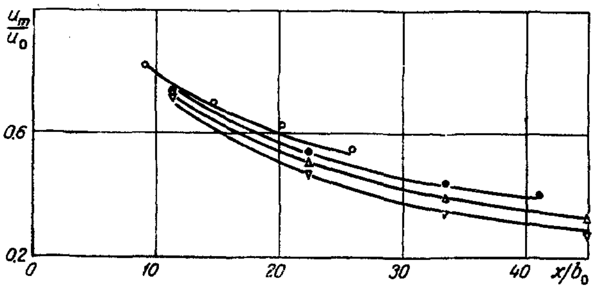

Fig. 3. Variation of maximum velocity along surface. Curves based on Eq. (11) (1—R = 60; 2—80; 3—120) and a plane surface (4).

After transformations of the same type as before, we obtain

$$Q = 2\pi r u_m \left[\int_0^{\delta_m} \left(1 - 0.18 \log \frac{y}{\delta_m} \right) dy + \right.$$

$$\left. + \int_{\delta_m}^{\infty} \mathrm{sch}^2\, 0.88 \frac{y - \delta_m}{\delta}\, dy \right] f(R,\, r) =$$

$$= 2\pi u_m \left(\frac{0.0736\, R}{R + 23.4}\, x + \frac{0.837\, R}{R - 5}\, b_0 \right) R \sin 57.3\, \frac{r}{R}. \tag{16}$$

Since the initial rate of flow from the annular nozzle

$$Q_0 = 2\pi r_0 b_0 u_0,$$

using expression (11), we find the equation for the change of flow rate in any section of the fan jet for flow over plane and concave surfaces as a fraction of the initial flow rate:

$$q = \frac{Q}{Q_0} = (1.19)$$

$$\left[\left(\frac{0.0736\, R}{R + 23.4}\, x + \frac{0.837\, R}{R - 5}\, b_0 \right) R \sin 57.3\, \frac{r}{R} \right]^{1/2} \times$$

$$\times (b_0 r_0)^{-1/2}$$

$$= 1.41\, \frac{u_0}{u_m}. \tag{17}$$

An analysis shows that as the radius of curvature of the surface decreases, the apparent mass also falls.

The results of Schwarz and Cosart [3], who detected a weak dependence of the characteristics of a plane semibounded jet on the Reynolds number at the nozzle exit when the latter was varied over a broad range, suggest that our principal results for a semibounded fan jet are also valid over a broad range of Reynolds numbers.

NOTATION

u_0 is the mean exit velocity; u is the variable velocity in the jet cross section; u_m is the maximum velocity in the jet cross section; δ_m is the thickness of the inner region, or the distance from the surface to a point at which the longitudinal velocity component has a maximum; δ is the conditional thickness of the outer region, or the distance from a point with maximum velocity to a point at which the velocity is equal to half the maximum; b is the width of the semibounded fan jet; b_0 is the nozzle gap; r_0 is the nozzle radius; R is the radius of curvature of the concave surface; x is the distance from the nozzle exit to the section in question, measured along the arc of the concave spherical surface, or along the radius of the plane surface; $r = x + r_0$; T_0 is the excess temperature at the nozzle exit; ΔT is the variable excess temperature in the jet cross section; ΔT_m is the maximum excess temperature in the jet cross section; Q is the volume fluid flow rate in an arbitrary section of the jet; Q_0 is the initial rate of flow from the nozzle.

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7 August 1967

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